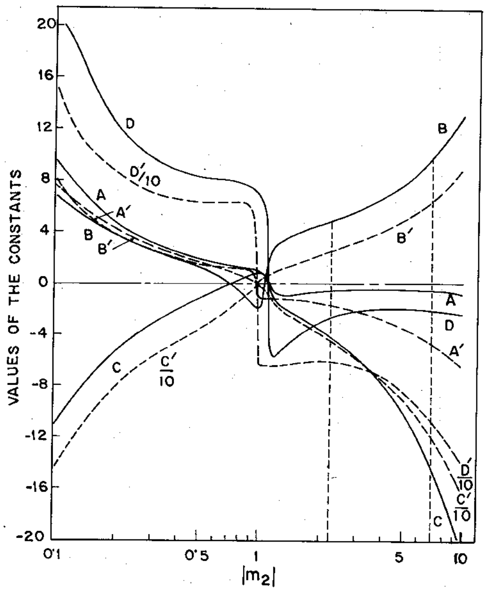

Fig. 2. Plot of error coefficients against normalized input impedance. Solid curves for $\sigma = 10$ mho/m and dotted curves for $\sigma = 100$ mho/m.

$$D = \frac{4}{N\beta_0^2K}\{(\beta_s^2 - \alpha_s^2)\cdot\cos\theta\cdot(1-\rho_2^2) - (\beta_s^2\tan\phi + \alpha_s^2\cot\phi)\cdot\sin\theta\cdot(1+\rho_2^2)\}$$

$$N = \{(1+\rho_2^2)^2 - 4\rho_2^2\cos^2\theta\}$$

$$\beta_0^2 = \omega^2\mu_0\epsilon_0$$

and $d\theta$ and $d\rho_2$ are the errors in the measurement of phase and magnitude of ρ_2.

For measurements using a transformer a small amount of error also creeps through the uncertainty in the value of Γ_d and p. As pointed out earlier, the error in p is nil for an ideal lossless transformer. With the calibration technique employed in the experiment, it is possible to measure the angle of the reflection coefficient of the shorted transformer with an accuracy better than 0.3°. Numerical computation indicates that for the experimental transformers with thickness round about $\lambda_g/4$ this phase angle changes at the rate of nearly 1° for a change of thickness equivalent to 0.005 rad. This shows that the error in Γ_d hardly exceeds 0.1 percent.

To test the dependence of the accuracy of measurement on the choice of the transformer material, we have computed in the case of silicon the coefficients of (4a) and (4b) as a function of $|m_2|$ for two conductivities 10 mho/m and 100 mho/m. The results are shown in Fig. 2. It appears that the best accuracy is attainable when $|m_2|$ is nearly unity, a condition which produces minimum VSWR in the input guide. The condition is rather stringent in cases of measurement on highly conducting samples. The choice of the dielectric constant for the transformer material under optimum condition would therefore be governed by the well-known matching relation

$$|\Gamma_d|^2 = |\Gamma_s||\Gamma_1|. \tag{5}$$

We can employ the curves of Fig. 2 to test the relative accuracies of the conventional and the modified technique. For our sample the value of $|m_2|$ is nearly 7 for the conventional measurement, while the same is nearly 2.2 using the Teflon transformer. Obviously, there is an improvement in the accuracy by a factor of nearly 3 in the measurement of K and by a factor of 2 in σ, the overall accuracy in the measurement of K and σ being better than 4 percent for 1-percent accuracy in the measurement of ρ_2.

Finally, we conclude that the application of a quarter-wave transformer brings about an improvement in the accuracy of semiconductor parameter determination in reflection-type measurement and also enables one to overcome the error associated with this type of measurement due to imperfect sample geometry.

Acknowledgment

The authors wish to thank Prof. B. R. Nag for his encouragement and helpful discussions, and the authorities of the Indian Statistical Institute for computer facilities.

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Factors Limiting the Signal-to-Noise Ratio of Negative-Conductance Amplifiers and Oscillators in FM/FDM Communications Systems

A. A. SWEET

***Abstract*—A derivation is presented for the signal-to-noise ratio of negative-conductance amplifiers and oscillators in FM/frequency division multiplexing (FDM) communications applications. Results indicate the limiting value of signal-to-noise ratio depends on the semiconductor properties and channel loading only. This means circuit adjustments, such as *Q*, cannot increase the signal-to-noise ratio without bounds. Typical specifications are given. Limiting values of signal-to-noise ratio for Gunn and Si IMPATT devices are given in typical applications. Results indicate that Gunn devices have a clear advantage over Si IMPATT's in a signal-to-noise sense.**

I. Introduction

IMPATT and Gunn devices are on the threshold of finding wide application in communications systems applications. However, certain fundamental limits may restrict the usefulness of these devices in some applications. The goodness of a communications system is measured in terms of its information capacity. In practice this reduces to a measure of the single-channel signal-to-noise ratio for a given number of information carrying channels. It is the intent of this short paper to expose those factors which limit the signal-to-noise ratio of negative-conductance amplifiers and oscillators.

II. Signal-to-Noise Ratio of a Negative-Conductance Reflection Amplifier

Any amplifier has a white noise output N_{out} which may be expressed in terms of its noise figure F as

$$N_{out} = FGkTB \tag{1}$$

where

- G amplifier power gain;
- k Boltzmann's constant $1.38 \times 10^{-23} J/K$;
- B measurement bandwidth;
- T 300 K.

It is customary to refer this noise to the amplifier's input by dividing N_{out} by G. In the presence of a signal, white noise power is equally divided between FM and AM sidebands [1]. The FM noise-to-carrier ratio contribution of the amplifier is

$$\mathrm{N/C} = \frac{FkTB}{2P_{in}} \tag{2}$$

where P_{in} is the power input to the amplifier.

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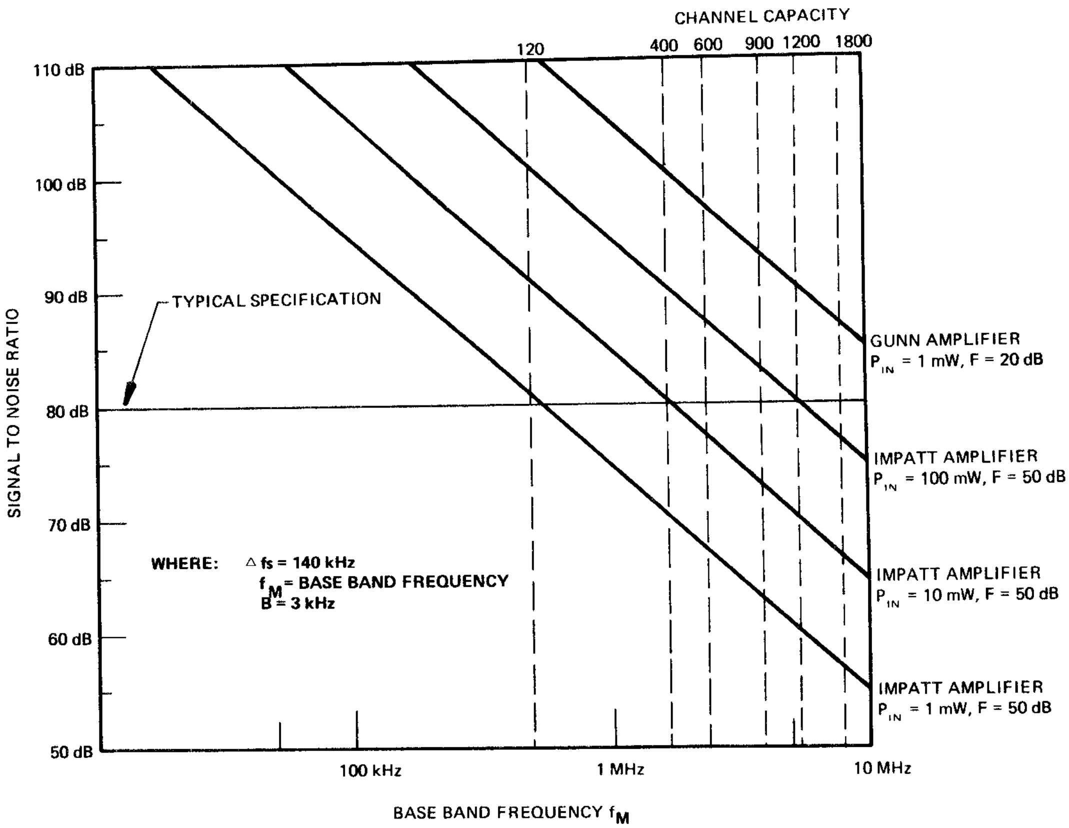

Fig. 1. Amplifier signal-to-noise ratio S/N $= (2P_{in}/FkTB)\cdot(\Delta f_s/f_M)^2$.

Now from small-signal FM theory it is well known [2] that in terms of rms frequency deviation, $\overline{\Delta f}_{rms}$, and baseband frequency fm, the FM noise-to-carrier ratio may be expressed as

$$\text{N/C} = \left(\frac{\overline{\Delta f}_{rms}}{fm}\right)^2. \tag{3}$$

By equating (2) with (3), an expression for the mean-square frequency deviation may be obtained as

$$\overline{\Delta f^2} = \frac{FkTB}{2P_{in}} fm^2. \tag{4}$$

The intrinsic signal-to-noise ratio of the amplifier is then obtained by dividing $\overline{\Delta f^2}$ by the square of the maximum single-channel deviation Δf_s. For the high-capacity system (1200 and 1800 channels) Δf_s = 140 KHz rms is customary. For the lower capacity systems Δf_s = 200 KHz rms is used:

$$\text{S/N} = \frac{\Delta f_s^2}{\overline{\Delta f^2}} = \frac{2P_{in}}{FkTB}\left(\frac{\Delta f_s}{fm}\right)^2. \tag{5}$$

Expression (5) tells us that as the baseband frequency increases, the signal-to-noise ratio decreases at 6 dB/octave. A frequency-division multiplex system occupies an amount of baseband in proportion to the number of multiplex channels. As the upper baseband frequency is pushed out to accommodate more channels, the value of S/N decreases. Channel bandwidth plus guard band is 4 KHz in frequency division multiplexing (FDM) telephony systems, so $(fm)_{max} = N \times 4$ KHz. For a fixed S/N specification, the number of channels which may be accommodated by a particular amplifier can be calculated from (5). Of course, S/N may be increased with a fixed noise figure by increasing P_{in}. This is the philosophy behind using a quiet preamplifier ahead of a noisy power amplifier. However, in high-capacity systems even preamplification to the 1-W level may not be sufficient if the power amplifier's noise figure exceeds 50 dB.

Fig. 1 shows a comparison of the calculated S/N for an F = 20-dB Gunn amplifier with that of an F = 50-dB Si IMPATT amplifier. These noise figures are typical of the respective devices, as discussed by Sweet [3], Perlman [4], and Thaler [5]. The IMPATT amplifier curves are extended to the cases of P_{in} = 1 mW, 10 mW, 100 mW, and 1 W. The Gunn amplifier is only evaluated at P_{in} = 1 mW. Dashed lines on Fig. 1 show the maximum baseband frequency used by the five different channel capacities being considered. The acceptable signal-to-noise ratio really depends on the noise budget of a particular system, however: 80 dB is a typical specification. The Gunn amplifier meets this specification easily with 1-mW input. The IMPATT amplifier with 1-mW input does not even meet this specification at 400 channels. In order to meet S/N = 80 dB for 1800 channels, the IMPATT amplifier will require noiseless preamplification to about 400 mW. Such preamplification may not even be sufficient since recent results have indicated that Si IMPATT effective noise figures may be over 60 dB at high power levels [5].

III. Signal-to-Noise Ratio of a Negative-Conductance FM Oscillator

Varactor-tuned IMPATT and Gunn oscillators are now being considered as directed transmitters of baseband information to radio frequencies. YIG-tuned oscillators may also be useful in these applications, but will not be considered in this analysis. The basic signal-to-noise ratio limitation in these oscillators comes about as follows. High baseband channel capacities require high electronic-tuning bandwidth. The aggragate deviation Δf_T of an FDM signal is given by

$$(\Delta f_T)^2 = N(\Delta f_s)^2 \tag{6}$$

where N is the number of channels and Δf_s is the maximum deviation per channel (140 KHz_{rms} for 1200 and 1800 channels).

The bandwidth of an electronically tuned oscillator is inversely proportional to its loaded Q. This means that if the channel capacity is increased, the oscillator's loaded Q must be decreased. It will be assumed that linearization techniques are employed so that the oscillator's full electronic-tuning bandwidth is useful (a large practical problem). A decrease in loaded Q means an increase in FM noise [2]. Since the single-channel maximum deviation Δf_s has not changed, the single-channel signal-to-noise ratio has been lowered.

An analysis of the signal-to-noise ratio must start with an expres-

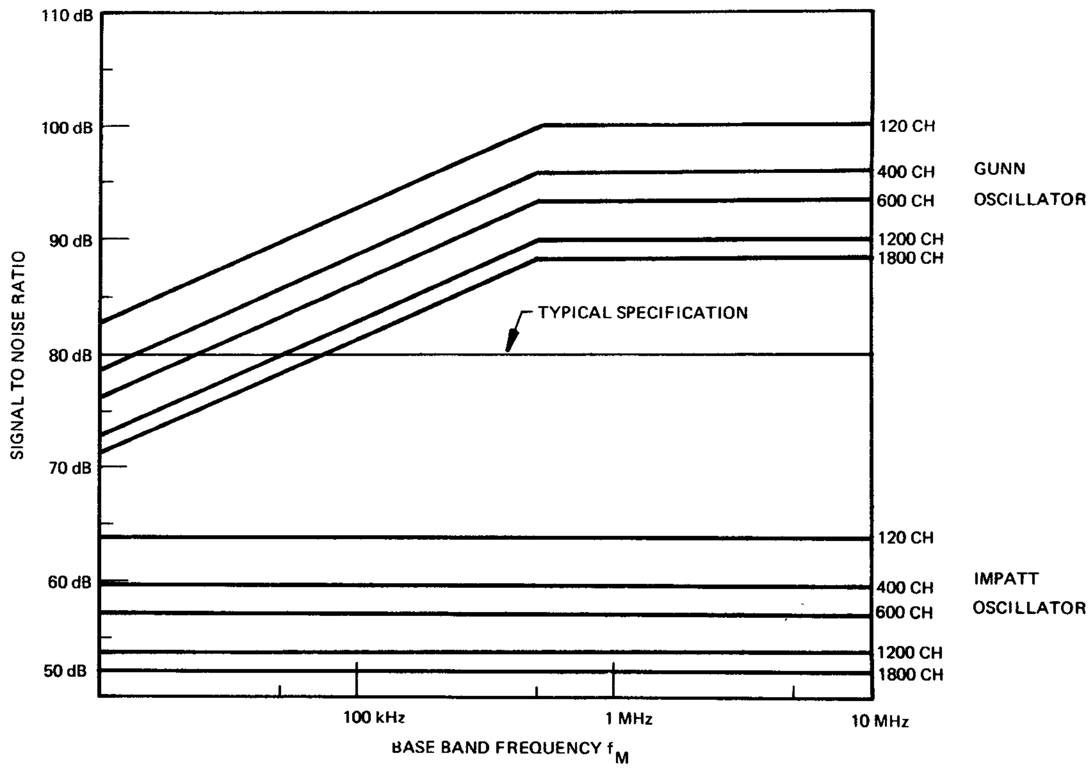

Fig. 2. Oscillator signal-to-noise ratio $f_0 = 6$ GHz, $P_0 = 5$ mW.

sion for FM noise. For Gunn oscillators, this expression has been given by Sweet [6] as

$$\overline{\Delta\omega^2} = \frac{\overline{i_{NS}^2}}{4C_T^2V_1^2} + \frac{\omega_0^2(\partial Cd/\partial V0)^2 S_{\Delta V0}(fm)B}{4C_T^2} \qquad (7)$$

where

$\overline{i_{NS}^2}$ quadrature phase component of the white noise source;
$S_{\Delta V0}(fm)$ equivalent low-frequency flicker noise spectrum;
C_T total circuit capacity, including the Gunn (or IMPATT) diode and varactor;
V_1 RF voltage across the Gunn (or IMPATT) diode;
fm baseband frequency;
ω_0 carrier frequency;
$(\partial Cd/\partial V0)$ Gunn diode's capacity bias sensitivity.

The first term on the right-hand side of (7) is the white noise component. In Si IMPATT oscillators this is the only noise component that is present [7]. The second term on the right-hand side of (7) is the flicker noise component. At high baseband frequencies, the white noise component will dominate, while at low baseband frequencies, the flicker noise component will dominate.

Now it can be shown [2] that for small-capacity variation (relative to the total capacity) the electronic-tuning bandwidth of a varactor-tuned oscillator is

$$\Delta\omega_T/\omega_0 = \Delta C/2C_T \qquad (8)$$

where $\Delta\omega_T$ is the tuning bandwidth and ΔC is the effective capacity variation imparted by the varactor to the cavity.

The signal-to-noise ratio is then simply found by dividing the square of (8) by (7):

$$\mathrm{S/N} = \frac{\Delta\omega_s^2}{\overline{\Delta\omega^2}} = \frac{\Delta\omega_T^2}{N\overline{\Delta\omega^2}} = \frac{\omega_0^2\Delta C^2/4C_T^2N}{\dfrac{\overline{i_{NS}^2}}{4C_T^2V_1^2} + \dfrac{\omega_0^2(\partial Cd/\partial V0)^2 S_{\Delta V0}(fm)B}{4C_T^2}}.$$

Notice that C_T (i.e., Q) may be cancelled out in the expression for S/N. Performing this rearrangement results in

$$\mathrm{S/N} = \frac{\Delta C^2/N}{\overline{i_{NS}^2}/\omega_0^2V_1^2 + (\partial Cd/\partial V0)^2 S_{\Delta V0}(fm)B}. \qquad (9)$$

Realizing that at high baseband frequencies (high slot) only white noise is present and at low baseband frequencies (low slot) only flicker noise is present; (9) may be broken up into two parts as

$$\mathrm{S/N}\,|_{\text{low slot}} = \frac{\Delta C^2}{N(\partial Cd/\partial V0)^2 S_{\Delta V0}(fm)B} \qquad (9a)$$

$$\mathrm{S/N}\,|_{\text{high slot}} = \frac{\omega_0^2V_1^2\Delta C^2}{N\overline{i_{NS}^2}}. \qquad (9b)$$

Equation (9b) may be simplified by introducing the relationships

$$P_0 = \frac{v_1^2\,|Gd|}{2}$$

$$\overline{i_{NS}^2} = 2kT_N\,|Gd|\,B$$

where $|Gd|$ is the magnitude of the diodes conductance and T_n is the noise temperature of the Gunn or IMPATT diode. These changes result in the final form for the high-slot signal-to-noise ratio:

$$\mathrm{S/N}\,|_{\text{high slot}} = \frac{\omega_0^2P_0\Delta C^2}{NkT_nB\,|Gd|^2}. \qquad (9b_1)$$

It is interesting to notice that in the low slot, the signal-to-noise ratio is entirely determined by the ratio of the information bearing varactor capacity variations to the random flicker noise capacity variations of the Gunn diode. Whereas in the high slot the signal-to-noise ratio may be improved by raising the carrier power and/or raising the operating frequency. The most important idea to be gleaned from these relationships is that the signal-to-noise ratio depends entirely on "device parameters." The usual degrees of circuit freedom such as Q and admittance cannot be employed to achieve a desired signal-to-noise ratio. In fact, only one circuit will produce the signal-to-noise ratio predicted by (9); all other circuits will only degrade it. This can be easily understood by first imagining the circuit Q is varied through a wide range of values. For very high values of Q, the electronic tuning will be insufficient to accommodate Δf_T. This will cause distortions which render the device useless. At low values of Q where the electronic tuning exceeds Δf_T, the FM noise will become excessive, decreasing S/N. This means that only one value of Q will produce the S/N predicted by (9). In a similar way only one value of circuit admittance will allow the device to realize its maximum generated power. High slot S/N depends on P_0 being maximized.

Consider some typical numbers. The intent will be to provide the best possible numbers that the present device technologies can provide in order to establish a limiting value of S/N. For now the value of ΔC must be derived from experience. A variety of practical consideration limit ΔC. They are package parasitic elements, consideration of resonance conditions which can lead to high losses, and local circuit transformations. It has been the author's experience with a wide variety of oscillator circuits that $\Delta C = 0.5$ pF is the practical upper limit. This number is a peak-to-peak value, converting to rms yields $\Delta C_{\rm rms} = 0.18$ pF.

The value of $(\partial Cd/\partial V0)$ for Gunn diodes has been found from experiment to be about 0.2 pF/V [8]. This number has been arrived at by two separate experiments. One experiment involved direct observation of the capacity of a stabilized device. The other experiment involved observing the pushing factor of an oscillator, and calculating $(\partial Cd/\partial V0)$ from a knowledge of the circuit Q and the device conductance.

A range of values for $S_{V0}(fm)$ has been given in a previous paper [6] If only the lowest noise devices are considered, an empirical expression for this spectrum in a 3-kHz bandwidth is

$$S_{\Delta V0}(fm)B = \frac{3 \times 10^{-7}}{fm} \text{ (V}^2\text{)}.$$

The noise temperature of a low-noise Gunn diode is 30 000 K [6]. Based on a noise figure of 50 dB, the noise temperature of an Si IMPATT is 3×10^7 K. $|-Gd|$ of a Gunn diode is approximately 10^{-2} mho for a 100 mW device.[1] The parallel equivalent circuit of an IMPATT diode also has a $|-Gd|$ of approximately 10^{-2} mho [8].

Fig. 2 shows a comparison of the signal-to-noise ratio of Gunn and IMPATT devices in a 50-mW pump application. As in the amplifier case, 120, 400, 600, 1200, and 1800 channels are considered. Customarily, 70 kHz is the lowest baseband slot. Notice that the Gunn oscillator can just meet the 80-dB specification at 70-kHz baseband for 1800 channels. Under the same conditions the IMPATT oscillator is nearly 30 dB out of spec. Even at low channel capacity the IMPATT oscillator is not close to the 80-dB specification, making IMPATT's look very unattractive for this application. It should be pointed out that since the high noise with Gunn devices occurs at low baseband frequencies, an advantage can be gained by moving the low slot frequency out in baseband.

[1] Unpublished work performed at the Monsanto Company, Microwave Production Group (now a part of Microwave Associates).

IV. Conclusions

1) Gunn amplifiers are capable of handling up to 1800 channels with inputs less than 1 mW.

2) Si IMPATT amplifiers require noiseless preamplification to several hundred milliwatts in order to handle 1800 channels.

3) Gunn oscillators can handle 1800 channels, although barely in the low slot.

4) Si IMPATT oscillators do not appear useful for carrying FM-FDM information.

5) GaAs IMPATT's, with noise properties midway between Si IMPATT's and Gunns may be more practical for 1800 channel amplifier service than Si IMPATT's. GaAs IMPATT's may also be capable of low-capacity oscillator service.

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Letters

Comments on "Wave Propagation on Nonuniform Transmission Lines"

SUHASH C. DUTTA ROY

***Abstract*—It is shown that the solutions for wave propagation on a nonuniform transmission line, recently proposed by Bergquist, are alternative forms of, or easily derivable from, the results of Protonotarios and Wing, given earlier.**

I. Introduction

In the above short paper,[1] Bergquist has proposed series solutions for the reflection coefficient, scattering parameters, and the admittance of a general nonuniform transmission line for arbitrary load conditions. The purpose of this letter is to show that these solutions are alternative forms of, or easily derivable from, the results of Protonotarios and Wing, given earlier in [1] and [2].

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[1] A. Bergquist, *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-20, pp. 557–558, Aug. 1972.

II. Protonotarios and Wing Formulas

Protonotarios and Wing formulas, generalized to an arbitrary nonuniform transmission line characterized by a series impedance per unit length $z(x)$ and a shunt admittance per unit length $y(x)$, with notations as in Fig. 1(a), are given by

$$\begin{bmatrix} v(0) \\ i(0) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v(l) \\ i(l) \end{bmatrix} \tag{1}$$

where

$$A = 1 + \int_0^l \int_0^{x_2} z(x_1)y(x_2)\, dx_1\, dx_2 + \int_0^l \int_0^{x_4} \int_0^{x_3} \int_0^{x_2} z(x_1)y(x_2)z(x_3)y(x_4)\, dx_1\, dx_2\, dx_3\, dx_4 + \cdots \tag{2}$$

$$B = \int_0^l z(x)\, dx + \int_0^l \int_0^{x_3} \int_0^{x_2} z(x_1)y(x_2)z(x_3)\, dx_1\, dx_2\, dx_3 + \int_0^l \int_0^{x_5} \int_0^{x_4} \int_0^{x_3} \int_0^{x_2} z(x_1)y(x_2)z(x_3) \cdot y(x_4)z(x_5)\, dx_1\, dx_2\, dx_3\, dx_4\, dx_5 + \cdots \tag{3}$$